

Test 3 Review

Dr. Graham-Squire, Spring 2012

- The test will cover sections 6.6 (tanks) and 8.1-8.5.
- To study, you should look over your notes, labs, rework HW problems, quizzes, and problems from the notes, as well as work out the practice problems given for each section. The Review Questions at the end of Chapter 8 will also be good practice (True/False (page 629) #1-9, Exercises # 1-34).
- Calculators can be used on this test, but will not be necessary.
- Some practice problems to work on:

1. A tank has the shape of an inverted circular cone with height 10 meters and base radius 4 meters. It is filled with water to a height of 8 meters. Note: The density of water is 1000 kg/cubic meter, and gravity is 9.8 m/sec²

(a) Find the work required to empty the tank by pumping all of the water to the top of the tank.

**ANS: 3, 362, 828 Joules

(b) Find the work required to pump the water to a point that is 6 meters above the top of the tank.

**ANS: 8, 407, 069 Joules

2. Determine whether the sequence is convergent or divergent. If convergent, find the limit.

(a) $a_n = \frac{n^3 + n^2 \cos n}{n^3}$

**ANS: Converges to 1. Split it into two fractions and use comparison.

(b) $b_n = \frac{\sqrt[3]{n}}{\ln n}$

**ANS: Diverges. Use L'Hospital's rule.

3. Determine if the series is convergent or divergent. If it is convergent, find the limit. Make sure you state which convergence/divergence test you use (or if no test, then explain your reasoning).

(a) $\sum_{n=1}^{\infty} \frac{3^n}{5^{n+2}}$

**ANS: Geometric series. Converges to 3/50.

(b) $\sum_{n=1}^{\infty} \frac{2}{n(n+2)}$ Note: You may need to use a partial fraction decomposition.

**ANS: Telescoping series. Converges to 3/2.

(c) $\sum_{n=1}^{\infty} \frac{3n^2 + 8}{(10n + 1)^2}$

**ANS: The limit of the terms approaches 3/100 \neq 0, so the series diverges by the test for divergence.

4. Test the series for convergence or divergence. If it converges, state whether it is absolutely convergent or not.

(a)
$$\sum_{n=2}^{\infty} \frac{(-5)^n - 7}{4^n}$$

**ANS: Diverges. Split it into two fractions, one is geometric with $|r| > 1$ so it diverges (the other converges, but it does not matter).

(b)
$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt[3]{n^4 + 10}}{n^2}$$

**ANS: Converges by the alternating series test, but is not absolutely convergent (to see it is not absolutely convergent you can do a limit comparison with $1/n^{(2/3)}$, which is a divergent p -series).

(c)
$$\sum_{n=1}^{\infty} (-1)^n \frac{n^4}{3^n}$$

**ANS: Use the ratio test, will be absolutely convergent.

5. (a) Find the sum of $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4}$ correct to five decimal places.

**ANS: Use the alternating series remainder test to see that you only need to add up the first ten terms, that is $1 - \frac{1}{16} + \frac{1}{81} - \dots$.

(b) How would you find the approximation if it was a 1 in the numerator instead of $(-1)^{n+1}$ (That is, how would you figure out how many terms you need to add up)?

**ANS: You would need to use the integral remainder test and find for what value of k you have $\int_k^{\infty} \frac{1}{x^4} dx$ less than 0.0001.

6. Find the radius of convergence and interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n(4^n)}$

**ANS: Use the ratio test to find that $R = 4$ and the interval is $[-6, 2)$.